POLE PLACEMENT TECHNIQUES USING STATIC AND DYNAMIC OUTPUT FEEDBACK

(Application to Power System Stabilizer Design)

A Thesis Submitted
in Partial Fulfilment of the Requirements
for the Degree of
MASTER OF TECHNOLOGY

By

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to the
DEPARTMENT OF ELECTICAL ENGINEERING
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AUGUST, 1981

CERTIFICATE

Certified that this work on "Pole Placement Techniques Using Static and Dynamic Output Feedback - Application to Power System Stabilizer Design" by S.S. Singh has been carried out under my supervision and that this has not been submitted elsewhere for a degree.

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ACKNOWLEDGEMENT

With a deep sense of gratitude I wish to express my indebtedness to Dr. S.S. Prabhu for his stimulating discussions, valuable suggestions and proper guidance throughout this thesis work.

I also wish to express my thanks to Mr. V.K. Sharma, Mr. H. Rao and Sqn. Ld. M. Murugasan for useful discussions and various other timely helps during this thesis work.

Thanks are also due to Mr. Abraham for skilful typing, to Mr. Dubey for efficient cyclostyling and to Mr. Shukla for photocopying.

S.S. Singh

ABSTRACT

The thesis considers the practical problem of power system stabilizer design.

An algorithm, which has not yet been used for design of power system stabilizers in the literature, has been used in this work. The algorithm involves partial and complete pole placement by output feedback.

The algorithm leads to the design of a full rank static or a full rank, minimal order dynamic compensator and shifts the poles arbitrarily closed to a specified set of pole locations. It has been applied to a practical power system and static and dynamic compensators have been designed.

The algorithm has been compared with the existing techniques for power system stabilizer design and advantages and disadvantages have been discussed.

Furthermore, the results of the two cases of partial and complete pole placement have been compared and merits and demerits have been discussed.

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LIST OF SYMBOLS

Symbol	Details
Δ	prefix to denote small changes about the operating point
o	subscript to denote the value at operating point
t	time in seconds
w_o	synchronous angular velocity in rad/sec.
w	instantaneous angular velocity in rad/sec.
δ	instantaneous rotor angle
v_t	machine terminal voltage
i	armature current
v_d, v_q	direct and quadrature components of v_t
i_d, i_q	direct and quadrature components of i
R	transmission line resistance
X	transmission line reactance
X_d	direct axis synchronous reactance
X_d'	direct axis transient reactance
T_{do}'	open circuit field time constant
H	inertia constant in seconds
T_e'	electromagnetic torque
T_m	mechanical torque input
x_1	state variable associated with voltage-regulator
T_R	voltage-regulator time constant
K_R	gain of voltage regulator
T_e	time constant of exciter system
E	voltage at the infinite bus system
I_a	rms value of i
I_d, I_q	Park's components of I_a
E_{fd}	correcting feedback signal from output of exciter

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CHAPTER I

INTRODUCTION

The problem of improvement in dynamic stability is a fundamental problem in Control Engineering. In modern power systems, dynamic stability problem has arisen due to the fast acting excitation control systems. Though a number of attempts have been made in the design of auxiliary controllers for excitation systems, known as 'Power System Stabilizers' and in many cases encouraging results have been obtained, they are either based on heuristic ideas or on trial and error methods. In this thesis, a systematic procedure has been applied for the design of power system stabilizers. This procedure is based on a recent work in pole-placement using output feedback [10].

The method used [10] is applicable to linear, time-invariant systems and is "full rank static or dynamic compensator design by output feedback" for achieving a given pole-placement.

The thesis is organized as follows :

In Chapter 2 only those parts of Munro and Hirbod's work, which are relevant from the application point of view, have been given. The details and proofs involved are given in part D

of Appendix A. Chapter 2 also presents a brief review of existing pole-placement techniques. Chapter 3 introduces power system stabilizers and reviews briefly the existing methods for their design. In Chapter 4 is presented a number of power system stabilizer designs for a particular power system. The results obtained have been discussed and compared with existing results. Chapter 5 presents conclusions.

CHAPTER 2

POLE-PLACEMENT TECHNIQUES

2.1 THE BASIC PRINCIPLE

The basic principle of pole-placement techniques is to shift the eigenvalues of the system to specified locations by state or output feedback.

2.2 REVIEW OF OLD TECHNIQUES

In the case of state feedback it has been shown that if the system is state controllable, then all the eigenvalues can be shifted to arbitrary locations provided complex eigenvalues exist in conjugate-pairs. If the system is not completely state controllable, then only those eigenvalues which are controllable, can be shifted to arbitrary locations by state feedback provided complex eigenvalues exist in conjugate pairs [2]. Many methods are available for pole-placement by state-feedback. However, in practice all the state variables may not be accessible. State reconstruction by state observers may be a solution of this problem. If 'l' represents the number of outputs, then the minimum order of the observer would be (n-l), where n represents the number of state variables.

Output feedback without attempting state reconstruction is a more practical approach. Even if a dynamic feedback is required, its order will, in general, be less than (n-l).

2.3 RECENT TECHNIQUES

Let us consider the linear, time-invariant system

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

where x is an n state vector, u is an m input vector and y is an l output vector. A, B, C are nxn, nxm and lxn constant state, input and output matrices respectively.

Davison [1] has shown that if the system is completely state controllable and observable and matrix A is cyclic, then at least max(m,l) poles can be shifted arbitrarily close to a given set of specified locations under the constraint that complex poles exist in conjugate pairs. The cyclicity of a matrix means that its Jordon normal cannonical form should not contain more than one jordon block for any eigenvalue. But the cyclicity of A is not a strong constraint, firstly because of being a ~~rare~~ common case and secondly because almost any arbitrary gain output feedback will make the system eigenvalues distinct and hence cyclic. This result is due to Davison [3]. The basic idea behind the placement of max(m,l) eigenvalues is dyadic feedback approach which is discussed in the next subsection.

2.3.1 Dyadic Feedback Approach

An output feedback law u = F.y is called dyadic feedback if F is of the form

$$F = f_c f_c^t ,$$

where f_c and f_o are vectors of order m and l respectively. Here, f_o^t represents transpose of f_o. It is this dyadic structure of output feedback that leads to simple linear equations for solution if either f_c or f_o is assumed apriori, transforming the multivariable system into either a single input, multi-output system or a single-output, multi-input system. Any direct attempt to evaluate $m \times l$ parameters of a non-dyadic F or $(m+l-1)$ parameters of a dyadic F leads to the solution of non-linear equations which are quite involved even for simple cases.

So the basic procedure is to choose f_c, if $m \leq l$ or to choose f_o if $l < m$ without affecting observability and controllability and then to solve the n equations in the unknown elements of f_o or f_c as the case may be. Then the resulting feedback matrix places max(m,l) poles to the pre-assigned locations. The procedure is given in detail in part B of Appendix A.

The most obvious drawback of this static compensator is that rest of the poles, which are $[n-\max(m,l)]$ in number get placed at random. So dynamic compensator comes into picture as a result of attempts made to overcome this problem.

Chen and Hsu [4] have shown that for a single input, multi-output system a compensator of order $r = (\nu_o - 1)$ is

sufficient to place all (n+r) poles of the composite system, where 'ν_o' is the observability index of the system. They have also shown that a compensator of order $r=(\nu_c-1)$ is sufficient to place all (n+r) poles of the composite system in the case of a multi-input, single output system, where 'ν_c' is controllability index of the system.

Brasch and Pearson [5] have shown that a dynamic compensator of order $r = \min(\nu_c-1, \nu_o-1)$ is sufficient to place all (n+r) poles of composite system to specified locations where 'ν_c' and 'ν_o' are controllability and observability indices respectively. The feedback used by Brasch and Pearson [5] is also of dyadic type, but proof and method are different than ~~that~~ those of Chen and Hsu [4] and more involved too.

Subsequently, the work of Ahmeri and Vacroux [6] on dynamic compensators with dyadic output feedback ~~showed~~ shows that $\max(\alpha+r, \beta+r)$ poles can be placed in an arbitrary set of pre-assigned locations, where r is the order of the compensator and

$$\alpha = \text{Rank}\,[B, AB, A^2B, \ldots, A^rB]$$

$$\text{and } \beta = \text{Rank}\,[C^t, A^tC^t, (A^t)^2C^t, \ldots, (A^t)^r C^t]$$

Then, come the works using feedback as a sum of dyads and it becomes vident that earlier works are only sufficient conditions for multi-variable systems. Subsequent works by

Davison and Wang [7], Belletrutti [8], Munro and Hirbod [10] show that more poles than max(m,l) can be assigned with static compensator and a lesser order of dynamic compensator than min(ν_c-1, ν_o-1) can place all of the (n+r) poles of the composite system.

2.3.2 Two Dyad Approach

Davison and Wang [7] have shown that with a sequence of two dyads min(n, m+l-1) poles can be placed arbitrarily close to a given set of pre-specified locations in 'almost all' cases under the constraints that complex poles exist in conjugate pairs and system is completely state controllable and observable.

The basic idea behind feedback by a sequence of two dyads is pole-retention as well as pole-placement. Through the first dyad, max(m,l) poles are placed just as in the single dyad case. Then during second dyadic feedback f_c or f_o is chosen to make [max(m,l)-1] poles either uncontrollable or unobservable. Then equations are solved for f_o or f_c of the second dyad to place as many more poles as possible. Davison and Wang [7] have shown that ~~during~~ in 'almost all' cases min(n,m+l-1) - [max(m,l)-1] poles can be placed during this second dyadic feedback. This leads to the placement of min(n,m+l-1) poles in almost all cases through a set of two dyads. But this static compensator still possess the old

drawback of placing the remaining $n-(m+l-1)$ poles randomly if $n > m+l-1$.

To overcome this drawback dynamic compensators again come into picture but with the new approach of two dyad feedback. It turns out that a lower order compensator than the previous case of single dyad feedback is sufficient to place all the $(n+r)$ poles of the composite system and this order r will lie in between

$$\left[\frac{n-(m+l-1)}{\max(m,l)}\right] \quad \text{and} \quad [n-(m+l-1)] \;,$$

where $[\cdot]$ represents the least integer, but greater than or equal to the enclosed quantity.

Similarly in the case of partial pole placement of $(q+r)$ poles [If $n \geqslant q > m+l-1$] of the composite system, where 'r' is the compensator order, it turns out that in 'almost all' cases, r will lie in between

$$\left[\frac{q-(m+l-1)}{\max(m,l)}\right] \quad \text{and} \quad [q-(m+l-1)] \;,$$

where $[\cdot]$ represents least integer, but greater than or equal to the enclosed quantity. But in this case of partial pole placement the unassigned $(n-q)$ poles will get placed randomly.

2.3.3 Extension of Two Dyad Approach

Further extension of the two dyad sequence approach has been made [8-10] but increase in the number of dyads beyond two has not resulted in lesser order dynamic compensator for complete pole placement. Similarly, increase in the number of dyads beyond two has not resulted in assignment of more poles than min(n,m+l-1) for the static compensator case.

But Belletrutti [8] has recently shown that increase in the rank of feedback compensator results in lowering of eigenvalue sensitivity with respect to parameter variations. This has been confirmed by Gomathi [9] through a numerical example.

2.3.4 Full Rank Compensator by a Minimal Sequence of Dyads

Munro and Hirbod [10] have proposed a new algorithm for full rank compensator design by a minimal sequence of dyadic feedbacks. In essence, it is an extension of pole-retention and pole-placement technique. It possess all the advantages of earlier techniques besides its own. Since every feedback is of dyadic form, it preserves the mathematical simplicity of linear equations. Since it is of full rank, eigenvalue sensitivity, due to parameter variations, is less.

Furthermore, since the feedback is an extension of two ~~composed of maximal~~ dyad technique ~~sequence of dyads~~, so it results in a minimal order dynamic compensator. with respect to all those dynamic compensators which are determined by a sequence of dyades.

The basic idea in Munro and Hirbod's algorithm for the case of $m \leqslant l$ is given below.

In the first step $f_c^{(1)}$ is chosen as $[1,0,0, \ldots, 0]^t$ and $f_o^{(1)}$ is found to place as many poles as possible.

In the second step $f_c^{(2)} = [f_1^2, f_2^2, 0, \ldots, 0]^t$; $f_2^2 \neq 0$ is chosen to make maximum of the assigned poles in first step uncontrollable. Then $f_o^{(2)}$ is found to place as many more poles as possible to specified locations.

This process is repeated until the (m-1)th time. Each time $f_c^{(i)} = [f_1^i, f_2^i, \ldots, f_i^i, 0, \ldots 0]^t$; $i = 3, \ldots, (m-1)$; $f_i^i \neq 0$; is chosen to make maximum of the previously assigned poles uncontrollable and then $f_o^{(i)}$ is found to place as many more poles as possible to specified locations.

In the mth step $f_c^{(m)} = [f_1^m, f_2^m, \ldots, f_m^m]$; $f_m^m \neq 0$: is chosen to retain (m-1) previously assigned poles and then $f_o^{(m)}$ is found to place the remaining poles.

If number of poles to be assigned is $q \leqslant (m+l-1)$, then a constant $f_o^{(m)}$ will result and F will be static. If $n \geqslant q > m+l-1$, the resulting $f_o^{(m)}$ will be dynamic and F will also become dynamic.

This method is given in detail in part D of Appendix A.

For the case of $m > l$, its dual is applicable.

This method is quite suitable for partial as well as complete pole-placement and in each case the method results in a minimal order, full rank compensator. In case of *$q \leq m+l-1$, the resulting compensator is static in 'almost all' cases.

Though in the case of partial pole placement the unspecified poles get placed randomly, it may so happen that they may get acceptable locations. In such cases, partial pole-placement is at an advantage in terms of lower order compensator but with good system performance.

But the cases, in which locations of unspecified poles are not acceptable, complete pole-placement can always be done for proper pole-placement. However, possibility of obtaining a good design with partial pole-placement should always be explored first.

In the present thesis, design of compensators have been done for both the cases, namely, partial and complete pole placement. In many cases of partial pole-placement results are very good and in a few cases even static compensators have resulted. But in a few cases poles have got placed at odd locations too. The complete pole-placement has always resulted in satisfactory results.

CHAPTER 3

POWER SYSTEM STABILIZERS

3.1 INTRODUCTION

Attempts have been made to improve the transient stability of ~~the~~ power systems by incorporating fast acting, high gain excitation systems with synchronous machines. Although this leads to ~~the~~ improvement in transient stability of the system, it often gives rise to the problem of oscillations and dynamic instability depending upon loading conditions [17-19]. Oscillations of sustained or growing nature have been reported in the literature [18-21]. The basic cause of such oscillations is the high gain of the voltage regulator, which alongwith the fast acting static-exciter introduces negative damping in the system affecting its dynamic stability [16]. So a need for another supplementary unit, to be used in the excitation control arises which can improve the dynamic stability of the system [17]. This supplementary unit is termed as Power System Stabilizer (PSS).

PSS is an auxiliary controller which receives a feedback signal from rotor angular position, angular velocity, frequency or acceleration and provides corrective signals at the input of the excitation system in order to damp out the oscillations in the system.

Since angular velocity, rotor angle, acceleration and frequency are directly measurable variables, only these are used as feedback signals.

Since dynamic stability of a power system is related to the changes in the system response due to small disturbance in the system, the PSS can be designed with respect to the linearized model of power system about an operating point. Clearly, pole-placement techniques using output feedback can be used for PSS design.

3.2 PSS DESIGN OBJECTIVE

As discussed in Section 3.1, the reason of instability in power system is negative damping introduced by the high gain excitation control system for system voltage regulation.

Since in a linear system negative damping can be viewed as being due to positive real part of a complex eigenvalue, the objective of PSS design will be to shift the unstable complex eigenvalues to appropriate locations in left half plane. This will make the system stable.

3.3 A BRIEF SURVEY OF EXISTING PSS DESIGN TECHNIQUES

Initially classical control theory [15,16] was applied to get improved system response. But it was limited to single-input, single-output systems.

Later on, modern control theory was applied. Here two

main approaches are available, namely, that based on linear optimal regulator theory [22,23] and that based on pole-placement techniques [9,24,25].

Application of linear optimal regulator theory results in a state feedback. However, a drawback of this approach is that all states have to be accessible for measurement. Furthermore it is an iterative technique in which one has to keep on changing the weighting matrices Q and R in the performance index till satisfactory response is obtained.

Among the pole-placement techniques, state feedback has been used for PSS design [24,25,28]. But it is not a practical case, because all of the state variables are not available for measurement in power systems.

Among output feedback pole-placement techniques [9], there are two well known approaches, namely, algebraic output feedback and dynamic output feedback.

In the case of algebraic feedback, a dynamic compensator with defined structure and parameter is chosen first. The choice of compensator structure and parameters usually depend on experience and heuristic ideas. After choosing this compensator the algebraic feedback law is found to place the poles to appropriate locations [9].

In the case of dynamic feedback, too, a compensator structure of some fixed order is assumed first and then the

parameters of this compensator are calculated to place the poles to appropriate locations [9].

In all of the above methods some drawbacks are always present. They are either based on heuristic ideas or on trial and error methods or they suffer from the preassumption on compensator structure and order or from inaccessibility of states. No straightforward method, which can directly give the order, the structure and the parameter values of PSS has been used so far.

In this thesis such a method has been applied for PSS design. The algorithm used in the method is due to Munro and Hirbod [10]. The algorithm is applicable to partial as well as complete pole-placement ~~apart~~ in addition to its ~~from the~~ advantages mentioned above. In the case of partial pole-placement the compensator will turn out a static one if the number of poles to be assigned is less than or equal to (m+l-1), where m and l are the number of inputs and outputs respectively.

CHAPTER 4

POWER SYSTEM STABILIZER DESIGN

4.1 INTRODUCTION

It has already been shown in Chapter 3 that PSS can be designed with respect to the linearized model of power system, at the operating point of interest, for improvement in dynamic stability. So in Section 4.2, a linearized state-space model of a synchronous generator connected to an infinite bus through a transmission line has been chosen. Derivation of this linearized model is given in Appendix B. In Section 4.3, the problem of PSS design has been formulated in precise terms. In Section 4.4, the algorithm to be used for PSS design is given. In Section 4.5 numerical results have been presented and in Section 4.6 the results have been discussed.

4.2 POWER SYSTEM MODEL

The power system, whose dynamic behaviour is to be improved by output feedback, consists of a synchronous generator connected to infinite bus through a transmission line alongwith exciter-voltage regulator as given in Figs. 4.1 - 4.3. Appendix B gives the nonlinear equations governing the system dynamics and the linearized equations derived from the non-linear equations about an operating point. The linearized system equations are

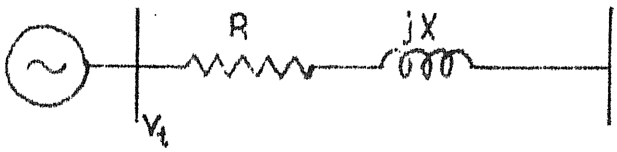

Fig. 4.1 System diagram

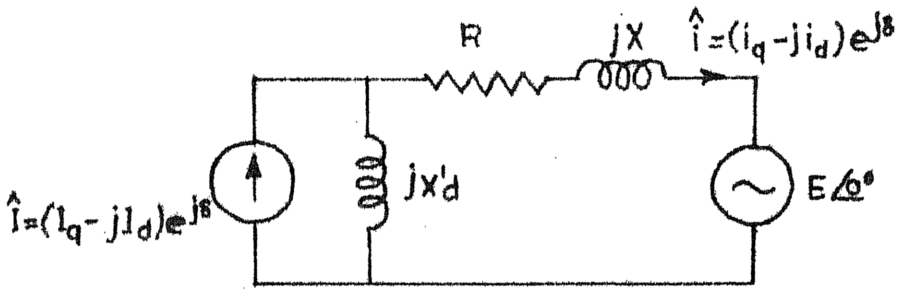

Fig. 4.2 Single phase equivalent circuit of the model

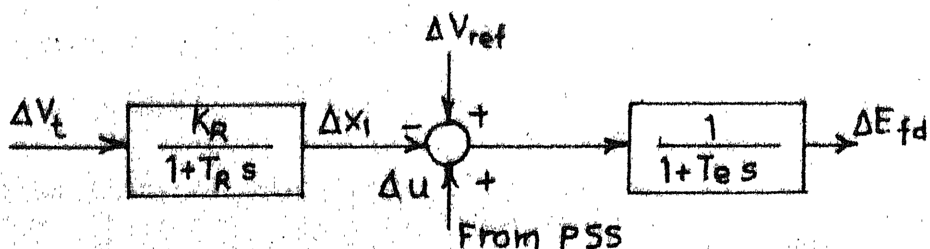

Fig. 4.3 Block diagram of excitation system

$$\dot{x} = Ax + Bv$$

$$y = Cx$$

where $x^t = [\Delta I_d, \Delta\delta, \Delta w, \Delta x_1]$ and $v = [\Delta u]$. Here x is an n state vector and v is a scalar input. y is a scalar/vector output depending upon the number of outputs. A,B,C are constant matrices of appropriate dimensions. A and B are given below :

$$A = \begin{bmatrix} -0.16312 & -0.3773 & 0.0 & -0.34722 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ -9.0999 & -25.7124 & 0.0 & 0.0 \\ 2983.2839 & -2436.8389 & 0.0 & -100.0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.34722 \\ 0.0 \\ 0.0 \\ 0.0 \end{bmatrix}$$

There are three different cases of output feedback which have been examined. They differ in the choice of output variables. The three cases are as follows :

a) Feedback from angular velocity (Δw) only

b) Feedback from angular velocity (Δw) and angular position ($\Delta\delta$), and

c) Feedback from angular acceleration ($\Delta\dot{w}$) and angular velocity (Δw).

Only the above-mentioned variables are considered for feedback, since only these can be directly and easily measured. In all the three cases matrix C and output y are different and are given below :

Case (a) $C = [0.0,\ 0.0,\ 1.0,\ 0.0]$ and $y = [y_1] = [\Delta w]$

Case (b) $C = \begin{bmatrix} 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 \end{bmatrix}$ and $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \Delta w \\ \Delta\delta \end{bmatrix}$

Case (c) $C = \begin{bmatrix} -9.0999 & -25.7124 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \end{bmatrix}$ and

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \Delta \dot{w} \\ \Delta w \end{bmatrix}$$

For all the three cases, partial as well as complete pole-placement have been attempted. Eigenvalues of the system without PSS is given in Table 4.1. It is seen that dominant eigenvalues are unstable while the remaining two are stable.

Table 4.1 Eigenvalues of the power system without PSS

S.No.	Eigenvalues
1.	0.2632367 + j5.587844
2.	0.2632367 - j5.587844
3.	-12.46339
4.	-88.2262

So the main design objective while designing PSS is to shift the real part of the dominant eigenvalues to a location in negative half plane such that system with PSS becomes stable.

The new locations for the dominant poles have been chosen as $-1.0 \pm j5.5878$. So, the minimum number of poles to be placed is 2. Now different possible cases of partial and complete pole-placement are given below :

Case I : Partial pole placement : Since n = 4, hence different possible cases are (a) q = 2, and (b) q = 3, where q poles are to be placed.

Case (a) q = 2. The specified locations are given below.

(i) $-1.0 + j5.5878$

(ii) $-1.0 - j5.5878$

Case (b) q = 3. The specified pole locations are given below.

(i) $-1.0 + j5.5878$

(ii) $-1.0 - j5.5878$

(iii) -12.46339

Case II : Complete pole-placement : In this case q = n = 4, and the specified pole locations are given below :

(i) $-1.0 + j5.5878$

(ii) $-1.0 - j5.5878$

(iii) -12.46339

(iv) -88.2262

4.3 PROBLEM FORMULATION

There are three cases and in each case the problem formulation is different. These cases arise due to the partial or complete pole-placement and due to static or dynamic type of compensator. Let q be the number of poles to be placed and m and l be the number of inputs and outputs respectively. Then in the present power system case $n > m+l-1$.

Case I : <u>$q \leq m+l-1$: partial pole-placement with static compensator</u>

The problem is to find an output feedback law

$$u = -F.y$$

where F is a constant matrix, such that q poles of the original system get placed at the specified locations.

Case II : <u>$n > q > m+l-1$: partial pole-placement with dynamic compensator</u>

The problem is to find an output feedback law

$$U(S) = -F(S)\,Y(S)$$

where F(S) is a proper rational polynomial matrix of minimal order r, such that (q+r) poles of the composite system get placed at the specified locations.

Case III : $n = q \geq m+l-1$: complete pole-placement with dynamic compensator

The problem is to find an output feedback law

$$U(s) = -F(s) \cdot Y(s)$$

where F(s) is a proper rational polynomial matrix of minimal order r, such that all the (n+r) poles of the composite system get placed at specified locations.

4.4 ALGORITHM

To determine a F or F(s) as required in Section 4.3, the algorithm of Munro and Hirbod has been used. Necessary details and proofs are given in part D of Appendix A.

4.4.1 Preliminary

This algorithm leads to the determination of a non-unique but full rank feedback matrix F and places the eigenvalues of the closed-loop system arbitrarily close to the specified locations. First of all the case of $n \leq m+l-1$ has been considered.

The desired matrix F can be constructed by a minimal sequence of dyads, that is,

$$F = \sum_{i=1}^{\mu} f_c^{(i)} f_o^{(i)}$$

where $f_c^{(i)}$ is an $m\times 1$ vector, $f_o^{(i)}$ is an $1\times l$ vector and $\mu = \min(m,l)$ The matrix F can equally be expressed as a

product of two matrices

$$F = F_c F_o$$

where columns of F_c are the vectors $f_c^{(i)}$ and row of F_o are vectors $f_o^{(i)}$, for $i = (1, 2, \ldots, \mu)$ in both cases. Now the algorithm is given below for the case $m \leqslant l$. For $l < m$, the dual is applicable.

4.4.2 Algorithm for the case $m \leqslant l$

Following are the m stages in the algorithm.

(1) Select $f_c^{(1)}$ as $[1, 0, \ldots, 0]^t$, and determine the corresponding $f_o^{(1)}$ to place as many pole as possible in desired locations.

(2) For $f_c^{(2)} = [f_1^2, f_2^2, 0, \ldots, 0]^t$, determine f_1^1 and f_2^2; $f_2^2 \neq 0$ to retain as many previously assigned poles as possible and determine $f_o^{(2)}$ to place at least one more pole in a desired location.

(3) For $f_c^{(3)} = [f_1^3, f_2^3, f_3^3, 0, \ldots, 0]^t$, determine f_1^3, f_2^3, f_3^3; $f_3^3 \neq 0$, to retain as many previously assigned poles as possible and determine $f_o^{(3)}$ to place at least one more pole in a desired location.

Repeat the sequence till $f_c^{(m-1)}$ and $f_o^{(m-1)}$ are determined.

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(m) For $f_c^{(m)} = [f_1^m, f_2^m, \ldots, f_m^m]$, determine f_1^m to f_m^m; $f_m^m \neq 0$, to retain $(m-1)$ previously assigned poles and then

determine $f_o^{(m)}$ to place min(l,n-m+1) poles in desired locations which is possible in 'almost all' cases [7].

So all the n poles have been placed for the case $q = n \leq m+l-1$. In case of $n \geq q > m+l-1$, the first (m-1) stages are carried out in accordance with the sequence given above and in the mth stage $f_c^{(m)}$ is determined as usual and then all the remaining poles (yet not placed) are placed in according with the method of Chen and Hsu [4], which is given in part A of Appendix A.

4.5 RESULTS

The results are given in Tables 4.3 - 4. 5 .

Unit step responses of rotor angle and angular velocity with respect to time have been given in Figs. 4.4 and 4.5 respectively for all cases except two. These two cases correspond to those partial pole-placement cases in which the unassigned poles have got placed in positive half plane leading to instability. The various controller configurations for which responses have been plotted are given in Table 4.2.

Different parameters corresponding to various cases have been defined below :

Case (a) : The case of partial pole-placement :

Here input-output relationship is given as

$$u = -F.y$$

Table 4.2

Various Controller Configurations

S.No.	Type of compensator and feedback variables
1.	Static compensator; Feedback from $\Delta\dot{w}$ and Δw
2.	Static compensator; Feedback from Δw and $\Delta\delta$
3.	Dynamic compensator of order 1; Feedback from Δw only
4.	Dynamic compensator of order 1; Feedback from $\Delta\dot{w}$ and Δw
5.	Dynamic compensator of order 2; Feedback from $\Delta\dot{w}$ and Δw
6.	Dynamic compensator of order 2; Feedback from Δw and $\Delta\delta$
7.	Dynamic compensator of order 3; Feedback from Δw only

where $F = [f_1, f_2]$ and f_1 and f_2 are constants.

Case (b) : The case of complete pole-placement :

(1) The case of $l = 1$; In this case input-output relationship is as follows

$$U(s) = -F(s) \cdot Y(s)$$

where $$F(s) = \frac{\beta_0 s^r + \beta_1 s^{r-1} + \ldots + \beta_r}{s^r + \alpha_1 s^{r-1} + \ldots + \alpha_r}$$

where r is the order of compensator. So for the first order compensator, it is of the form

$$F(s) = \frac{\beta_o s + \beta_1}{s + \alpha_1} \quad ,$$

for the second order compensator form, it is of the form

$$F(s) = \frac{\beta_o s^2 + \beta_1 s + \beta_2}{s^2 + \alpha_1 s + \alpha_2}$$

and for the third order compensator, it is of the form

$$F(s) = \frac{\beta_o s^3 + \beta_1 s^2 + \beta_2 s + \beta_3}{s^3 + \alpha_1 s^2 + \alpha_2 s + \alpha_3}$$

(ii) The case of $l = 2$: The input-output relationship

$$U(s) = -F(s) \cdot Y(s)$$

where $F(s)$ is of the form

$$F(s) = \frac{1}{\Delta(s)} [\Gamma_1(s), \quad \Gamma_2(s)]$$

with $\Delta(s) \qquad = s^r + \alpha_1 s^{r-1} + \alpha_2 s^{r-2} + \ldots + \alpha_r$

and $\Gamma_i(s) \qquad = \beta_{oi} s^r + \beta_{1i} s^{r-1} + \beta_{2i} s^{r-2} + \ldots + \beta_{ri}$

for $i = 1, 2$.

There are two cases of r and they are as follows.

For $r = 1$

$$F(s) = \frac{1}{s+\alpha_1} [\beta_{o1} s + \beta_{11}, \quad \beta_{o2} s + \beta_{12}]$$

Table 4.3
Static Compensator (Partial Pole Placement)

S. No.	1	2
Feedback variables	Δw & $\Delta \delta$	$\Delta \dot{w}$ & Δw
No. of poles to be assigned (q)	2	2
Assigned locations	$-1.0 \pm j5.5878$	$-1.0 \pm j5.5878$
No. of poles left for random placement(n-q)	2	2
Values of the compensator parameters	$f_1 = -7.60$ $f_2 = 19.84$	$f_1 = -8.83$ $f_2 = -0.62$
Location of the assigned poles in closed loop system	$-1.000 \pm j5.5878$	$-1.000 \pm j5.5878$
Location of the unassigned poles in closed loop system	-88.28, -9.894	-87.97, -12.14

Table 4.4

Dynamic Compensator (Partial Pole Placement)

S.No.	1	2	3	4
o. of poles o be placed q+r)	q=2 r=1 q+r=3	q=3 r=1 q+r=4	q=3 r=1 q+r=4	q=3 r=2 q+r=5
re-assigned ocations	1.0±j5.5878, 12.46339	-1.0±j5.5878, -12.46339, -88.2262	-1.0±j5.5878, -12.46339, -88.2262	-1.0±j5.5878 -12.46339, -88.2262 -20.00
eedback ariables	Δw	Δw & $\Delta\delta$	$\Delta\dot{w}$ & Δw	Δw
ompensator arameters	$\beta_0=-68.6$ $\beta_1=-855.01$ $\alpha_1=99.092406$	$\beta_{01}=0.0$, $\beta_{02}=11.24$ $\beta_{11}=11.24$ $\beta_{12}=278.06$ $\alpha_1=1.5669$	$\beta_{01}=-0.37$, $\beta_{02}=0.0$, $\beta_{11}=-37.27$ $\beta_{12}=-407.05$ $\alpha_1=47.014$	$\beta_0=6.38$ $\beta_1=624.24$ $\beta_2=7013.42$ $\alpha_1=-15.2316$ $\alpha_2=-777.6149$
ocation of ssigned oles	-1.000001 ±j5.5877, -12.46339	-0.9967508 ±j5.60701, -12.46104, -88.22741	-1.000001 ±j5.5878, -12.46339, -88.2262	-0.999999 ±j5.58779, -12.46363, -88.22621, -19.99979
ocation of nassigned oles	-85.55061, -99.24153	0.9518578	-45.644459	37.75814

Note :- 'r' represents compensator order.

Table 4.5
Dynamic Compensator (Complete Pole Placement)

S. No.	1	2	3
Feed back variables	Δw & $\Delta\delta$	$\Delta\dot{w}$ & Δw	Δw only
No. of poles to be placed q=n, r, n+r	n=q=4 r=2 n+r=6	n= =4 r=2 n+r=6	n=q=4 r=3 n+r=7
Preassigned locations	-1.0±j5.5878 -12.46339 -88.2262 -20.00 &-15.00	-1.0±j5.5878 -12.46339, -88.2262, -20.00 & -15.00	-1.0±j5.5878, -12.46339, -88.2262, -20.00, -15.00 & -25.00
Order of compensator	r=2	r=2	r=3
Compensator parameters	β_{01}=-2.46, β_{02}=0.0, β_{11}=-240.67, β_{12}= 0.0, β_{21}=-1970.51, β_{22}=8057.78 α_1 =-37.52647, α_2 =382.90734	β_{01}=-0.23, β_{02}=0.0, β_{11}=25.92, β_{12}=0.0, β_{21}=-493.07, β_{22}=-2572.79, α_1 =36.786575, α_2=308.91761	β_0= -8.32 β_1= -88.60 β_2= -14297.32 β_3= -56262.04 α_1 = 62.52647 α_2 = 1302.5721 α_3 = 7722.9402
Locations of assigned poles	-1.000002 ±j5.5878, -12.46341, -88.22621, -20.0001, -14.99997	-1.0±j5.5878, -12.46339, -88.2262, -19.99999, & -15.00001	-0.9999847 ±j5.587785, -12.46311, -88.2262, -19.99886, -15.00101, -25.00045

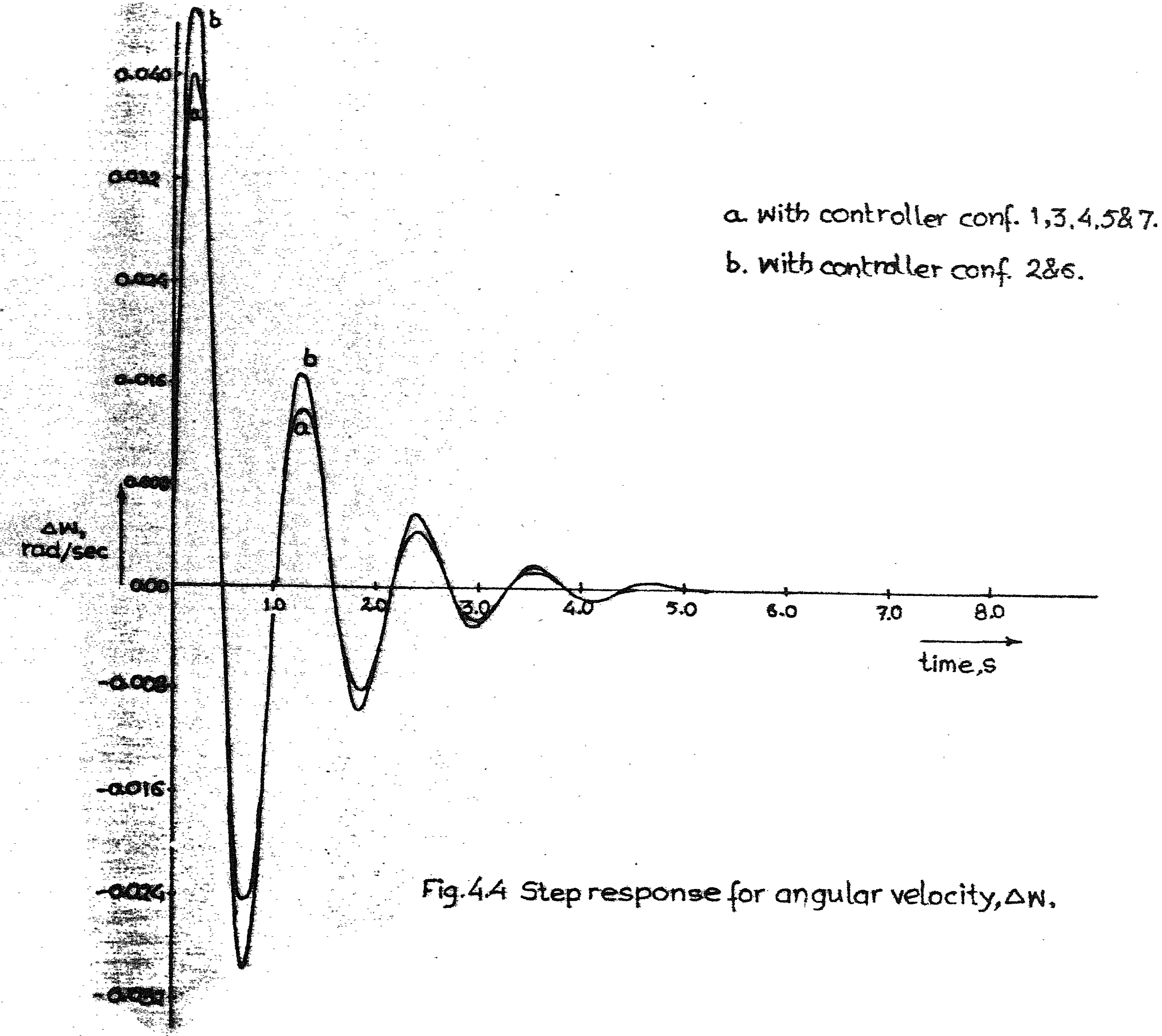

Fig.4.4 Step response for angular velocity, ΔW.

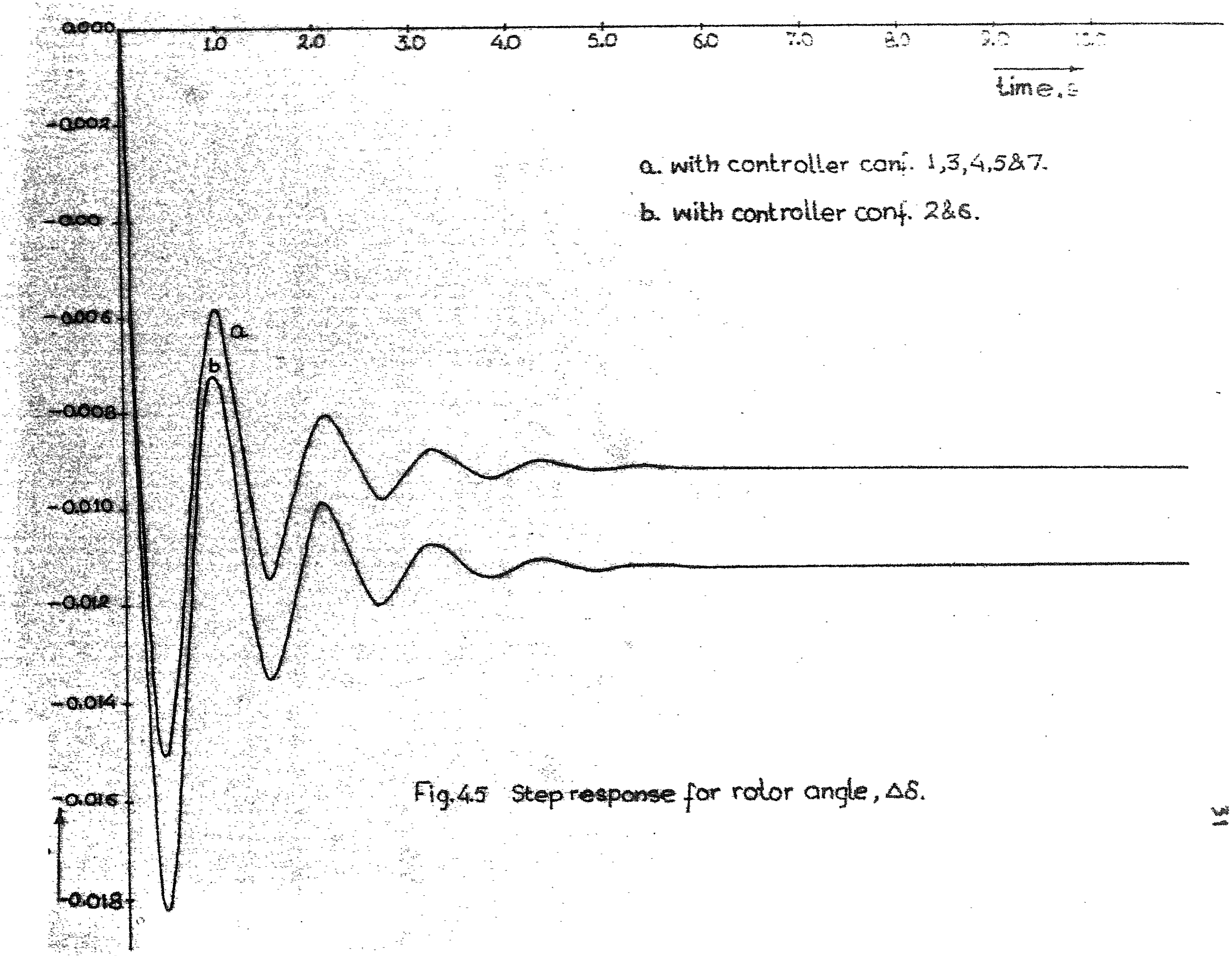

Fig.4.5 Step response for rotor angle, $\Delta\delta$.

and for r = 2

$$F(s) = \frac{1}{s^2+\alpha_1 s+\alpha_2} [\beta_{01}s^2 + \beta_{11}s + \beta_{21}, \beta_{02}s^2 + \beta_{12}s + \beta_{22}]$$

The values of different parameters of F as well as F(s) are given alongwith ~~in~~ the tables 4.2 to 4.5.

4.5 DISCUSSION

After a careful inspection of the results, it appears that Δw is the most effective feedback control element among all the three measurable variables, namely, $\Delta\delta$, Δw and $\Delta\dot{w}$.

Furthermore, it also appears that a combination of Δw and $\Delta\dot{w}$ as feedback elements is a best choice.

The case of partial pole-placement reveals that the unspecified poles can get placed anywhere which is in accordance with our expectation. But in most of the cases they have got placed at good locations and in a few cases at extremely good locations, as in the case of Δw feedback for placement of 2 poles, and in the case of $\Delta\dot{w}$ and Δw feedback for placement of 3 poles.

Static compensators have resulted in the cases of Δw and $\Delta\delta$ feedback as well as $\Delta\dot{w}$ and Δw feedback for placement of two poles and the unspecified poles have also got placed at reasonably good locations.

Hence attempts for partial pole-placement are not fruitless.

As far as the complete pole-placement is concerned, it is found that the pole-placement is more or less exact. Furthermore in the complete pole-placement case there is a certainty about the improvement in system dynamic response because there is no chance of getting any pole placed at an undesirable location.

As far as the algorithm is concerned, it is simple and straightforward and places the poles quite accurately. Furthermore, this algorithm itself determines the minimal order and structure of the compensator which is the most important advantage of this algorithm over the other existing ones.

A study of the responses given in Figs. 4.4 and 4.5 reveals that there is no distinct advantage of the dynamic compensators over the static ones, because the responses in all the cases are almost identical. In the present case it is so because new poles and zeros, originated due to dynamic type of compensator, are too close and cancel the effect of each-other. But this type of behaviour is not guaranteed in all cases. In general, all the three cases, mentioned below can take place.

(a) New zero is nearer to dominant pole than the new pole.

(b) New zero and new pole are nearly equidistance from the dominant pole.

(c) New zero is at greater distance with respect to new pole from the dominant pole.

Since, the dominant poles are placed at the same locations in dynamic as well as static compensators and original zeros do not shift by output feedback in the case of single-input, multi-output or multi-input, single-output system. The response of a dynamic compensator can differ ~~than~~ from that of static compensator only due to the effect of these newly added pole-zero locations.

Suppose new zero and pole are at z_n and p_n respectively and the dominant pole is at p_d. Then the response corresponding to the most dominant pole of the system will be

$$\frac{p_d - z_n}{p_d - p_n} \; K \, e^{p_d t} \, ,$$

where $K\,e^{p_d t}$ is the response due to the dominant pole of the composite system with static compensator. Hence the response of the dynamic compensator will be better than static one, if

$$\frac{p_d - z_n}{p_d - p_n} < 1 \, .$$

So, only in case (a), the dynamic compensator will result in better response than static one. In case (b) the response will be nearly identical, which is the present case too and in case (c) response of dynamic compensator will be worse than the static one.

Hence, dynamic compensator will be advantageous over static one if new zero is nearer to dominant pole than the new

pole. But it can not be guaranteed that the new zero will always be nearer to dominant pole than the new pole.

Hence it can be concluded that, it is not necessary that the dynamic compensator will give better response than the static compensator if the unspecified poles of the latter got placed at acceptable locations.

So, the superiority of the dynamic compensator with complete pole-placement over lower order dynamic or static compensator can be claimed in only those cases where the unspecified poles of latter system do not get placed at acceptable locations.

CHAPTER 5

CONCLUSION

In the present work a new algorithm has been used for PSS design. This algorithm is better than those which have been used earlier for PSS design in many respects, besides being simple, straightforward and computationally easy. It is neither based on any heuristic procedure nor involves trial and error procedures. No apriori assumptions need be made about compensator structure. Apart from these, the algorithm directly gives the minimum possible order and structure of compensator whereas earlier works, dealing with PSS design, involve assumptions regarding the order and structure of the compensator.

Besides, presenting a new algorithm to the PSS design area, some other conclusions have also been reached regarding the suitability of a dynamic compensator. It has been concluded through ~~logic and~~ example that it is quite fruitful to attempt partial pole-placement and it is quite likely that the unspecified pole may get placed at acceptable locations.

It has also been shown that if a lower order dynamic compensator (due to partial pole-placement) exists, then it is not necessary that the system with higher order dynamic compensator (due to complete pole-placement) will always have

better response than that with lower order. In general the response may worsen, may remain same or may improve. It all depends upon the newly added poles and zeros.

It has also been concluded that dynamic compensator with complete pole-placement is better than the lower order dynamic or static compensators with incomplete pole-placement only if the unspecified poles of the latter cases do not get placed at acceptable locations.

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APPENDIX A

VARIOUS OUTPUT FEEDBACK APPROACHES

GENERAL

Let a linear, time-invariant system be given by

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

where x is an n-state vector, u is an m-input vector and y is an l-output vector and A,B,C are constant matrices of order nxn, nxm and lxn respectively.

Now this system can be represented in the frequency domain by the transfer function G(s) as

$$G(s) = C(sI-A)^{-1}B = \frac{C \; Adj(sI-A)B}{det(sI-A)} = \frac{\Gamma(s)}{\Delta(s)}$$

and

$$Y(s) = G(s)\,U(s).$$

where $\Delta(s) = det\,(sI-A)$ is the characteristic polynomial of the system and its zeros $(\lambda_1, \lambda_2, \ldots, \lambda_n)$ are the poles of the system.

Now our problem is to determine a feedback law of the form

$$U(s) = R(s) - F(s)\, Y(s)$$

which assigns the (n+r) poles of the composite system to specified locations $(\bar{\lambda}_1, \ldots, \bar{\lambda}_n)$.

Here, R(s) is a vector of r reference inputs and r is the order of F(s), the compensator.

Now it is well-known that any direct attempt to evaluate the parameters of F(s) results in a set of non-linear equations It is equally well known that if F is dyadic, that is, F is defined as the outer product of two vectors f_c and f_o^t as

$$F = f_c\, f_o^t \qquad (G1)$$

where $f_c \in R^m$, $f_o \in R^l$, then a direct attempt to solve (m+l-1) independent parameters defining F also requires the solution of a set of non-linear equations. The usual approach to this latter problem is to choose either f_c or f_o^t without affecting controllability and observability. It is this choice of f_c or f_o^t that makes the remaining part of the system design linear.

PART A

DYADIC COMPENSATOR DESIGN (SINGLE-INPUT, MULTI-OUTPUT CASE) [4]

Consider a system described by an irreducible lx1 strictly-proper rational transfer function matrix G(s), defined as

$$G(s) = \frac{1}{\Delta_o(s)} \begin{bmatrix} N_1(s) \\ \vdots \\ N_l(s) \end{bmatrix} = \frac{\Gamma_o(s)}{\Delta_o(s)}$$

where $\Delta_o(s) = s^n + \alpha_1 s^{n-1} + \dots + \alpha_n$

and

$$N_i(s) = \beta_{i1} s^{n-1} + \beta_{i2} s^{n-2} + \dots + \beta_{in}, \quad (i = 1, \dots, l).$$

$\Delta_o(s)$ is the least common denominator of the elements of $G(s)$, and, therefore, is alsothe characteristic polynomial of $G(s)$.

The problem is to determine a 1xl dynamic feedback compensator $F(s)$ having least degree such that the closed-loop system

$$H(s) = [I + G(s)\,F(s)]^{-1}\,G(s)$$

has a desired set of poles ($\tilde{\lambda}_i$; $i = 1, 2, \dots, n+r$).

Now by assuming $F(s)$ of the form

$$F(s) = \frac{1}{\Delta_c(s)} [M_1(s), \dots, M_l(s)] = \frac{\Gamma_c(s)}{\Delta_c(s)}$$

where

$$\Delta_c(s) = s^r + h_1 s^{r-1} + \dots + h_r$$

and

$$M_i(s) = g_{i0} s^r + g_{i1} s^{r-1} + g_{i2} s^{r-2} + \dots + g_{ir}$$

$$i = 1, 2, \dots, l.$$

the resulting closed loop polynomial $\Delta_d(s)$, defined as

$$\Delta_d(s) = s^{n+r} + d_1 s^{n+r-1} + d_2 s^{n+r-2} + \dots + d_{n+r}$$

can be written as

$$\Delta_d(s) = \Delta_o(s)\,\Delta_c(s) + \sum_{i=1}^{1} N_i(s)\,M_i(s) \qquad \text{(A1)}$$

The above result can be proved as follows :

$$H(s) = \frac{\Gamma_d(s)}{\Delta_d(s)} = [I + G(s)\,F(s)]^{-1}\,G(s)$$

$$= G(s)\,[I+F(s)\,G(s)]^{-1}$$

$$= \frac{G(s)\,\mathrm{Adj}[I + F(s)\,G(s)]}{\det\,[I+F(s)\,G(s)]}$$

But F(s) G(s) is a 1x1 matrix, hence a simple rational polynomial and it is well known that adjoint of a single element matrix is 1. So,

$$H(s) = \frac{\Gamma_d(s)}{\Delta_d(s)} = \frac{G(s)}{\det[I+F(s)\,G(s)]} = \frac{G(s)}{1+F(s)\,G(s)}$$

$$= \frac{\dfrac{\Gamma_o(s)}{\Delta_o(s)}}{1+\dfrac{\Gamma_c(s)\,\Gamma_o(s)}{\Delta_c(s)\Delta_o(s)}} = \frac{\Delta_c(s)\,\Gamma_o(s)}{\Delta_c(s)\Delta_o(s)+\Gamma_c(s)\,\Gamma_o(s)}$$

Hence,

$$\Delta_d(s) = \Delta_c(s)\,\Delta_o(s) + \Gamma_c(s)\,\Gamma_o(s) = \Delta_c(s)\,\Delta_o(s) + \sum_{i=1}^{1} M_i(s)\,N_i(s)$$

Now by equating the coefficients of powers of s of eqn.(A1), a set of (n+r) equations in [(r+1)(l+1)-1] parameters of F(s) will be obtained which is given below :

$$X_r \; P_r = \delta_r \tag{A2}$$

where the vector P_r contains the parameters h_j's and g_{ij}'s of F(s) and the difference vector δ_r contains the coefficients δ_j of the polynomial

$$\delta_r(s) = \Delta_d(s) - \Delta_o(s)\, s^r.$$

Now, X_r, P_r and δ_r are given below :

$$X_r = \left[\begin{array}{cccc|cccc|c|ccc}
1 & 0 & \cdots & 0 & \beta_{11} & 0 & \cdots & 0 & \cdots & \beta_{11} & \cdots & 0 \\
\alpha_1 & 1 & \cdots & . & \beta_{12} & \beta_{11} & & 0 & \cdots & \beta_{12} & & . \\
\alpha_2 & \alpha_1 & & . & & \beta_{12} & & . & & & & . \\
 & & & 0 & & & & 0 & & & & 0 \\
 & & & 1 & & & & \beta_{11} & & & & \beta_{11} \\
 & & & \alpha_1 & & & & & & & & \\
 & & & \alpha_2 & & & & \beta_{12} & & & & \\
\alpha_{n-1} & & & & \beta_{1n} & \beta_{1n-1} & & & & \beta_{1n} & & \\
\alpha_n & \alpha_{n-1} & & & 0 & \beta_{1n} & & & & & & \\
0 & \alpha_n & & & & 0 & & & & & & \\
0 & 0 & & & & & & & & & & \\
\vdots & \vdots & & & & \vdots & & & & & & \\
0 & 0 & \cdots & \alpha_n & 0 & 0 & \cdots & \beta_{1n} & \cdots & 0 & & \beta_{1n}
\end{array}\right]$$

$$P_r = [h_1, \ldots, h_r, g_{10}, \ldots, g_{1r}, \ldots, g_{l0}, \ldots, g_{lr}]^t$$

and

$$\delta_r = \begin{bmatrix} d_1 \\ d_2 \\ \cdot \\ \cdot \\ \cdot \\ d_n \\ d_{n+1} \\ \cdot \\ \cdot \\ \cdot \\ d_{n+r} \end{bmatrix} - \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \cdot \\ \cdot \\ \cdot \\ \alpha_n \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}$$

Now, for eqn.(A2) to have solution, δ_r must lie in column space of X_r, i.e.

$$\text{rank}(X_r, \delta_r) = \text{rank}(X_r). \qquad (A3)$$

But, this condition will be satisfied, if the number of independent columns in X_r will be more or equal to (n+r), the number of rows.

From here the minimum possible value of r can be calculated. The condition for this will be

$$[(l+1)(r+1)-1] \geqslant n+r .$$

On simplifying,

$$r \geqslant [\tfrac{n-l}{l}] \qquad (A4)$$

where [K] implies the least integer $\geqslant$ K.

Similarly the upper limit can also be found with help of the results of Chen and Hsu [4] and Brash and Pearson [5] and it turns out that

$$r \leqslant [n-1]$$

Hence r is bounded as

$$[\frac{n-1}{l}] \leqslant r \leqslant [n-1], \qquad (A5)$$

where [K] implies the least integer $\geqslant$ K.

Here, one point should be noted that condition (A3) may get satisfied for $r < [\frac{n-1}{l}]$ in some special cases of δ_r. Hence, the best way to find minimum value of r is to follow the method given below :

a) Set $r = 0$

b) Check condition (A3). If satisfied, go to step d.

c) $r \leftarrow r+1$. Go to step b.

d) $r_{min} = r$. Stop.

After determining r by above method a solution for equation (A2) can be given by

$$P_r = [X_r^t X_r]^{-1} X_r^t \delta_r \text{ ; if } r < [\frac{n-1}{l}] ,$$

and

$$P_r = X_r^t [X_r X_r^t]^{-1} \delta_r \text{ ; if } r \geqslant [\frac{n-1}{l}] .$$

For the first case solution is unique, while for the second case it is one of the non-unique solutions.

PART B

DYADIC COMPENSATOR DESIGN (MULTIVARIABLE CASE)[1]

Here the procedure is to choose f_o^t; if $l < m$ or f_c; if $m \leq l$ (see equation G1), such that the resulting system is completely observable and controllable. The corresponding matrix X_o having dimension $n \times \max(m,l)$, will have full column rank $\max(m,l)$ and

$$P_o = [X_o^t\, X_o]^{-1}\, X_o^t\, \delta_o$$

will give the best solution for P_o in least error square sense. Furthermore, if δ_o lies in the column space of X_o, then the solution will be exact and unique.

Now, let $q < n$ be the number of closed-loop poles to be assigned, while $t = n-q$ poles are allowed to assume arbitrary values. The closed-loop characteristic polynomial $\Delta_d(s)$ for the present case can be written as

$$\Delta_d(s) = \Delta_q(s)\,\Delta_t(s)$$
$$= (s^2+d_1s^{q-1}+d_2s^{q-2}+\ldots+d_q)(s^t+e_1s^{t-1}+\ldots+e_t) \tag{B1}$$

and the difference vector δ_o in the present case will be redefined as

$$\delta_o = \hat{\delta}_o + e_1\delta_1 + e_2\delta_2 + \ldots + e_t\delta_t$$

where

$$\hat{\delta}_o(s) = \Delta_q(s)\cdot s^t - \Delta_o(s) \tag{B2}$$

and

$$\delta_i(s) = \Delta_q(s)\cdot s^{t-i} \quad , \quad i = 1, \ldots, t.$$

Then the equation (A2) becomes

$$X_o P_o = \hat{\delta}_o + D\ e \tag{B3}$$

where

$$D = [\delta_1, \delta_2, \ldots, \delta_t]$$

and $$e^t = [e_1, e_2, \ldots, e_t]$$

Here vector e^t is transpose of vector e. Eqn. (B3) can further be manipulated to ~~be~~ ~~in~~ the form

$$\hat{X}_o\ \hat{P}_o = \hat{\delta}_o \tag{B4}$$

where

$$\hat{X}_o = [X_o, -D] \quad \text{and} \quad \hat{P}_o^t = \begin{bmatrix} p_o \\ e \end{bmatrix}$$

Here dimension of $\hat{X}_o$ is $n\cdot$ [max)m,l)+t] = $n\cdot$[n-q+max(m,l)].

Now one of the following three cases may arise :

<u>Case I</u> : If q = max(m,l), then the dimension of $\hat{X}_o$ is $n\cdot(q+t)$ = nxn, and $\hat{P}_o$ can be uniquely determined as

$$\hat{P}_o = [\hat{X}_o]^{-1}\ \hat{\delta}_o \tag{B5}$$

<u>Case II</u> : If $q < \max(m,l)$, then number of columns $[n+\max(m,l)-q]$ will be more than number of rows and a number of solutions are possible. One of them is

$$\hat{P}_o = [\hat{X}_o^t \hat{X}_o]^{-1} \hat{X}_o^t \hat{\delta}_o \tag{B6}$$

<u>Case III</u> : If $q > \max(m,l)$, then the number of columns $[n-\{q+\max(m,l)\}]$ will be less than the number of rows, and a solution will exist, if $\hat{\delta}_o$ will lie in the column space of $\hat{X}_o$ and solution will be

$$\hat{P}_o = \hat{X}_o^t [\hat{X}_o \hat{X}_o^t]^{-1} \hat{\delta}_o \tag{B7}$$

If $\hat{\delta}_o$ does not lie in the column space of $\hat{X}_o$, then a dynamic compensator will be needed. In such a case, δ_r of eqn. (A2) should be redefined to give $\hat{\delta}_r$ as

$$\hat{\delta}_r(s) = \Delta_q(s)\, s^{t'} - \Delta_o(s)\, s^r \tag{B8}$$

and in this case $t' = n+r-q$. Under such condition equation (B4) will become

$$\hat{X}_r \hat{P}_r = \hat{\delta}_r \tag{B9}$$

The dimension of matrix $\hat{X}_r$ will now be $(n+r)\cdot[\max(m,l)\cdot(r+1)+r+t']$

Upper and lower bounds in the present case can be determined in the same way, as given in part A. It turns out that for placement of $(q+r)$ poles the compensator order r is bounded as

$$\left[\frac{q-\max(m,l)}{\max(m,l)}\right] \leq r \leq [q-\max(m,l)]$$

where $[K]$ implies the least integer $\geq K$.

PART C

COMPENSATOR DESIGN BY A SEQUENCE OF TWO DYADS [7]

In this the usual way is to assign as many poles as possible by a constant feedback matrix F_1 (first dyad) and then to assign the remaining poles by second dyad F_2 while retaining as many as possible poles assigned by feedback F_1.

So the desired compensator can be defined as

$$F = F_1 + F_2 \tag{C1}$$

where F_1 and F_2 both are dyadic.

F_1 is determined to place max(m,l) poles at desired location as already discussed in part B. Then, the remaining problem is to retain as many as possible of the max(m,l) poles, already assigned by the first dyad while attempting to shift some (or all) of the remaining [n-max(m,l)] poles to the desired locations.

Consider the case of $l > m$. Then using a feedback

$$F_1 = f_c^{(1)} f_o^{(1)} . \tag{C2}$$

let the resulting lxm system with l poles at the desired locations, be expressed as

$$H^{(1)}(s) = \Gamma^{(1)}(s)/ \Delta^{(1)}(s) \tag{C3}$$

We must now choose $f_o^{(2)}$ such that as many as possible of these l poles, say k in number, become unobservable, that is,

$$f_o^{(2)} \; \Gamma^{(1)}(s) = \Delta_k(s) \; N^{(2)}(s) \tag{C4}$$

Now let us find out what can be the maximum value of k.

The choice of $f_o^{(2)}$ is facilitated by the fact that for s equal to any of the roots of $\Delta^{(1)}(s)$ and in particular for s equal to any of the roots of $\Delta_k(s)$, the matrix $\Gamma^{(1)}(s)$ has rank equal to one.

So equation (C4) leads to a set of $k = \max(m,l)$ equations in $k = \max(m,l)$ unknown parameters of $f_o^{(2)}$:

$$f_o^{(2)} \; \Gamma^{(1)}(\lambda_i) = 0; \quad i = 1, \ldots, \max(m,l) . \tag{C5}$$

Taking only one row from each $\Gamma^{(1)}(\lambda_i)$ for $i = 1, \ldots, \max(m,l)$, eqn. (C5) can equally be written as

$$f_o^{(2)} \; \Gamma = 0 \tag{C6}$$

where ith row of Γ corresponds to any non-zero row of $\Gamma^{(1)}(\lambda_i)$.

Now dimension of Γ is max(m,l) $\times$ max(m,l). Eqn. (C6) has a non-trivial solution only if Γ has more rows than columns. This can be achieved only by assigning a value to one (or more) parameters of $f_o^{(2)}$. Thus, we can only restrain at the most [max(m,l)-1] poles out of the max(m,l) poles assigned by feedback F_1.

Having determined $f_o^{(2)}$ as above, we have $N_2(s)$ with dimension 1xmin(m,l), with elements having maximum degree [(n-1)-(max(m,l)-1)] = [n-max(m,l)]. Equation (B4) will now become

$$\hat{\hat{X}}_o \hat{\hat{P}}_o = \hat{\hat{\delta}}_o \qquad (C7)$$

where $\hat{\hat{\delta}}_o$ has dimensions min(m,n-l) $\cdot$1, $\hat{\hat{X}}_o$ has dimension min(m,n-l+1) $\cdot$m and $\hat{\hat{P}}_o = f_o^{(2)}$ has dimension mx1.

Now if n $\leq$ m+l-1, then n-l+1 $\geq$ m. So, the number of columns of $\hat{\hat{X}}_o$ will be greater than or equal to its number of rows, and (C7) will have a solution given by

$$\hat{\hat{P}}_o = [\hat{\hat{X}}_o]^{-1} \hat{\hat{\delta}}_o \qquad \text{if} \quad n = m+l-1$$

and

$$\hat{\hat{P}}_o = [\hat{\hat{X}}_o^t \hat{\hat{X}}_o]^{-1} \hat{\hat{X}}_o^t \hat{\hat{\delta}}_o \qquad \text{if} \quad n < m+l-1.$$

But if n > m+l-1, then eqn. (C7) will not have a solution unless $\hat{\hat{\delta}}_o$ is a special case such that it lies in the column space of $\hat{\hat{X}}_o$. In general, a dynamic compensator will be needed.

Applying eqn. (A5) shows that compensator order r will be bounded as

$$\left[\frac{\{n-(\max(m,l)-1)\} - \min(m,l)}{\max(m,l)}\right] \leq r \leq [\{n-(\max(m,l)\} - \min(m,l)]$$

or

$$\left[\frac{n-(m+l-1)}{\max(m,l)}\right] \leq r \leq [n-(m+l-1)]$$

for placement of all $(n+r)$ poles of closed loop system. Here $[K]$ implies the least integer $\geq K$.

It can also be verified that for placement of $(q+r)$ poles, where $n \leq q < m+l-1$, compensator order r will be bounded as

$$\left[\frac{q-(m+l-1)}{\max(m,l)}\right] \leq r \leq [q-(m+l-1)]$$

where $[\cdot]$ denotes least integer but greater than or equal to the enclosed quantity.

PART D

FULL RANK COMPENSATOR DESIGN BY MINIMAL SEQUENCE OF DYADS [10]

The procedure to be described in this part is essentially a generalization of two dyad feedback technique.

The desired feedback matrix F is constructed from a minimal sequence of dyads, that is,

$$F = \sum_{i=1}^{\mu} f_c^{(i)} f_o^{(i)} \qquad (D1)$$

where $f_c^{(i)}$ is an mx1 vector, $f_o^{(i)}$ is a 1xl vector, and $\mu = \min(m,l)$. The matrix F can equally be expressed as a product of two matrices as

$$F = F_c F_o, \qquad (D2)$$

where the columns of F_c are the vectors $f_c^{(i)}$ and the rows of F_o are the vectors $f_o^{(i)}$; $i = 1,2, \ldots, \mu$ in both cases. The matrix F will have maximum rank μ, if F_c and F_o both will have maximum rank.

In the following discussion, it will be seen that for the case $m \leqslant l$, F_c will have maximum rank by construction. It then remains to show that F_o will also have maximum rank. The result for the ease of $m > l$ is obtained by duality.

Essentially,at each stage in the proposed procedure, the vectors $f_c^{(i)}$ are chosen so that one more input to the original system is used. It is this action that guarantees the rank of F_c to be maximum. The vectors $f_o^{(i)}$ are then chosen to assign as many poles as possible at each stage.

The procedure and its proof is given below.

Let $f_c^{(1)} = [1,0, \ldots 0]^t$, say. Then, since it is assumed that B has rank m, the pseudo-input vector $b^{(1)} = Bf_c^{(1)}$ will influence at least one mode of the system $[A,b^{(1)},C]$. Let n_1; $1 \leqslant n_1 \leqslant n$; be the number of modes influenced

by $b_o^{(1)}$. The system $[A_1, b^{(1)}, C]$ has 1 outputs and 1 input. So now at the most q_1 poles can be assigned to specific locations at this stage, where $q_1 = \min(n_1, 1)$. Vector $f_o^{(1)}$ is then determined to place these poles at desired locations by solving

$$\Delta^{(1)}(s) = \Delta^{(o)}(s) + c^{(1)}\,[\operatorname{adj}(sI-A)]\,b^{(1)} \qquad \text{(D3)}$$

where $\Delta^{(1)}(s)$ is as defined by eqn. (B1), $\Delta^{(o)}(s)$ is the open loop characteristic polynomial $c^{(1)} = f_o^{(1)}C$, and $b^{(1)} = Bf_c^{(1)}$.

Now let $f_c^{(2)} = [f_1, f_2, \;, \ldots, 0]^t$, say, where $f_c^{(2)}$ is chosen such that as many as possible of the q_1 poles assigned in stage 1 are made uncontrollable with respect to system $[A^{(1)}, b^{(2)}, C]$, where $A^{(1)} = A - Bf_c^{(1)}\,f_o^{(1)}C$, and such that $f_c^{(2)} \neq 0$. This ensures that at least one mode of the system $[A^{(1)}, b^{(2)}, C]$ is controllable, and at least one mode is uncontrollable. Let n_2, such that $1 \leqslant n_2 \leqslant n-1$ be the number of modes influenced by $b^{(2)}$ and let $q_2 = \min(1, n_2)$ be the number of poles which can be further assigned to specific values at this stage. The vector $f_o^{(2)}$ is then determined to place q_2 poles at desired locations by solving

$$\Delta^{(2)}(s) = \Delta^{(1)}(s) + c^{(2)}[\operatorname{adj}(sI-A + b^{(1)}c^{(1)})]\,b^{(2)} \qquad \text{(D4)}$$

This procedure is repeated at each subsequent stage until at the mth stage the result of second part of Part C of this Appendix is applied.

It is easy to see that the matrix $F_c = f_c^{(i)}$; $i = 1, 2, \ldots, n$; generated in this manner ~~is~~ has full rank equal to m. The fact that F_o is also of full rank has been proved by Munro and Hirbod [10] and hence has been omitted here.

The procedure described here is the basis for the algorithm presented in Chapter 4.

APPENDIX B

MACHINE MODEL [9]

A three winding model of synchronous machine is considered. The stator of the machine is represented by a dependent current source. The power system network is represented by a single phase equivalent circuit as shown in Fig. 4.2.

The system non-linear equations are given below :

$$\frac{2H}{w_o} \frac{d^2\delta}{dt^2} = T_m - T_e' \qquad \text{(E1)}$$

$$\frac{dI_d}{dt} = [E_{fd} - X_d' I_d - (X_d - X_d') i_d]/(X_d' \, T_{do}') \qquad \text{(E2)}$$

and

$$I_q = \varepsilon \, i_q \qquad \text{(E3)}$$

where

$$T_e' = X_d' \, (I_d i_q - I_q i_d)$$

and

$$\varepsilon = (X_d' - X_q)/X_d'$$

The non-state variables i_d, i_q and I_q can be expressed in terms of δ and I_d as follows. From Fig. 2.2, the armature current phasor $\hat{i}$ can be solved for, by using the network equations

$$\hat{i} = \frac{jX_d'}{R+j(X+X_d')} \, I - \frac{E}{R+j(X+X_d')} = (a_1+ja_2)I - (y_1+jy_2)E \qquad \text{(B4)}$$

where

$$a_1 = \frac{X_d'(X+X_d')}{R^2+(X+X_d')^2}, \qquad a_2 = \frac{RX_d'}{R^2+(X+X_d')^2}$$

$$y_1 = \frac{R}{R^2+(X+X_d')^2} \quad \text{and} \quad y_2 = \frac{X+X_d'}{R^2+(X+X_d')^2}$$

So from (E4) and Fig. 2.2

$$i_q - ji_d = (a_1+ja_2)(I_q-jI_d) - (y_1+jy_2)E\,e^{-j\delta} \tag{E5}$$

By equating real and imaginary parts

$$i_q = a_1 I_q + a_2 I_d - E(y_1 \cos\delta + y_2 \sin\delta)$$

and

$$i_d = a_1 I_d - a_2 I_q + E(y_2 \cos\delta - y_1 \sin\delta)$$

Substituting I_q from eqns. (E3) in the above equation we get

$$i_q = \frac{a_2}{1-a_1\varepsilon} I_d - \frac{E}{1-a_1\varepsilon} (y_1 \cos\delta + y_2 \sin\delta) \tag{E6}$$

and

$$i_d = a_1 I_d - \frac{a_2\varepsilon}{1-a_1\varepsilon} [a_2 I_d - E(y_1 \cos\delta + y_2 \sin\delta)]$$

$$+ E(y_2 \cos\delta - y_1 \sin\delta) \tag{E7}$$

<u>Excitation System Equations</u>

From the block diagram of excitation system given in Fig. 4.3. The state equation can be written as

$$\frac{dx_1}{dt} = -\frac{x_1}{T_R} + \frac{K_R}{T_R}\ v_t \tag{E8}$$

and $E_{fd} = V_{ref} - x_1 + u$ (E9)

after neglecting the small time-constant T_e of static excitation. In eqns. (E8) and (E9)

$$v_t = [v_d^2 + v_q^2]^{\frac{1}{2}} = [(X_q\ i_q)^2 + X_d'(I_d - i_d)\ ^2]^{\frac{1}{2}} \tag{E10}$$

and u is the output of PSS.

So the non-linear differential equations can be written as

$$\dot{I}_d = [E_{fd} - X_d' I_d - (X_d - X_d')i_d]/(X_d'\ T_{do}') \tag{E11}$$

$$\dot{\delta} = w \tag{E12}$$

$$\dot{w} = \frac{w_o}{2H}[T_m - X_d'(I_d i_q - I_q i_d)] \tag{E13}$$

$$\dot{x}_1 = -\frac{x_1}{T_R} + \frac{K_R}{T_R} \tag{E14}$$

alongwith

$$E_{fd} = V_{ref} - x_1 + u$$

and i_q and i_d are already defined explicitly in terms of I_d and δ in (E6) and (E7).

<u>Linearized Model</u>

The linearized model can be given as

$$\begin{bmatrix} \Delta\dot{I}_d \\ \Delta\dot{\delta} \\ \Delta\dot{w} \\ \Delta\dot{X}_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 & a_{14} \\ 0 & 0 & 1 & 0 \\ a_{31} & a_{32} & 0 & 0 \\ a_{41} & a_{42} & 0 & a_{44} \end{bmatrix} \begin{bmatrix} \Delta I_d \\ \Delta\delta \\ \Delta w \\ \Delta X_1 \end{bmatrix} + \begin{bmatrix} b_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Delta u \qquad (E15)$$

where

$$a_{11} = -\frac{X_d' + (X_d - X_d')\, K_3}{X_d'\; T_{do}'}$$

$$a_{12} = -\frac{K_4(X_d - X_d')}{X_d'\; T_{do}'}$$

$$a_{14} = -\frac{1}{X_d'\; T_{do}'}$$

$$a_{31} = -\frac{w_o}{2H}\,(i_{qo} - K_3 I_{qo} + K_1 I_{do} - K_1 i_{do}\; q)X_d'$$

$$a_{32} = \frac{w_o}{2H}\,(K_4 I_{qo} - K_2 I_{do} + K_2\; i_{do}\; q)X_d'$$

$$a_{41} = \frac{K_R}{T_R}\left[\frac{v_{do}}{v_{to}}\,(RK_3 - XK_1) + \frac{v_{qo}}{v_{to}}\,(RK_1 + XK_3)\right]$$

$$a_{42} = \frac{K_R}{T_R}\left[\frac{v_{do}}{v_{to}}\,(E\cos\delta_o + RK_4 - XK_2)\right.$$

$$\left. + \frac{v_{qo}}{v_{to}}\,(-E\sin\delta_o + RK_2 + XK_4)\right]$$

$$a_{44} = -\frac{1}{T_R}$$

and

$$b_1 = \frac{1}{X_d'\; T_{do}'}$$

Here K_1, K_2, K_3 and K_4 are the constants which have been obtained after linearizing equations of i_q and i_d about the operating point.

So,

$$\Delta i_q = K_1 \Delta I_d + K_2 \Delta\delta$$

and

$$\Delta i_d = K_3 \Delta I_d + K_4 \Delta\delta$$

where

$$K_1 = \frac{a_2}{1-a_1\varepsilon}, \quad K_2 = \frac{E}{1-a_1\varepsilon}(y_1 \sin\delta_o - y_2 \cos\delta_o)$$

$$K_3 = a_1 - \frac{a_2^2 \varepsilon}{1-a_1\varepsilon} \text{ and } K_4 = \frac{E}{1-a_1\varepsilon}[-(a_2\varepsilon y_1 + (1-a_1\varepsilon)y_2)\sin\delta_o$$

$$+ (a_2\varepsilon y_2 - (1-a_1\varepsilon)y_1)\cos\delta_o]$$

The set of equations (E15) can be written in the following standard form

$$\dot{x} = Ax + bv$$

where $x^t = [\Delta I_d, \Delta\delta, \Delta w, \Delta x_1]$ is the 4th order state vector and $v = [\Delta u]$ is the scalar input which is the output of PSS too.

The matrices A and b are constant matrices at operating point, and their numerical values are given as

$$A = \begin{bmatrix} -0.16312 & -0.3773 & 0.0 & -0.34722 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ -9.0999 & -25.7124 & 0.0 & 0.0 \\ 1983.2893 & -2436.8389 & 0.0 & -100.0 \end{bmatrix}$$

$$b = \begin{bmatrix} 0.34722 \\ 0.0 \\ 0.0 \\ 0.0 \end{bmatrix}$$

System Data

Generator Prameters :

$X_d = 1.14, \quad X_q = 0.66, \quad X_d' = 0.24$

$T_{do}' = 12.0, \quad H = 1.5, \quad w_o = 314.0$

Network Parameters :

$R = 0.0, \quad X = 0.7$

Operating Data :

$E = 1.0 \angle 0^{\circ}$

P = Received power = 1.0

Q = Received reactive power = 0.0

T_R = Voltage regulator time constant=0.01 sec.